Why You Can't Ignore Those Vibration Fixture Resonances

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Vibration fixtures, at times, have resonant frequencies that exist in the frequency range needed for conducting vibration tests. These resonant frequencies can cause significant problems when running vibration tests over ranges which include these resonances. A test engineer will attempt to control the shaker system through a feedback accelerometer which controls the level of vibration input into the test specimen. However, the control accelerometer can only control the level of vibration at that point. It cannot change the resonant behavior of the vibration fixture. Some problems involving this are described in this article along with some basic material about these phenomena.

Often times, vibration tests, are performed to qualify or verify the acceptability of specific hardware for certain environments. We use shaker systems to generate forces or accelerations to replicate some known operating condition or to generate an input spectrum that envelops the actual environment. Usually, a specification mandates the particular input that is needed to satisfy the requirement.

The interface between the shaker and the test article is the test fixture. As defined in this article, the test fixture includes the shaker armature, expander head (or slip plate) as well as the attachment fixture (i.e., .the item designed to accommodate the test specimen). So, when we refer to the fixture, we mean everything between the shaker's driver coil and the test specimen. (Just because we can't see the armature, doesn't mean that it's not part of the fixture setup). Figure 1 shows a typical schematic of the elements of a fixture.

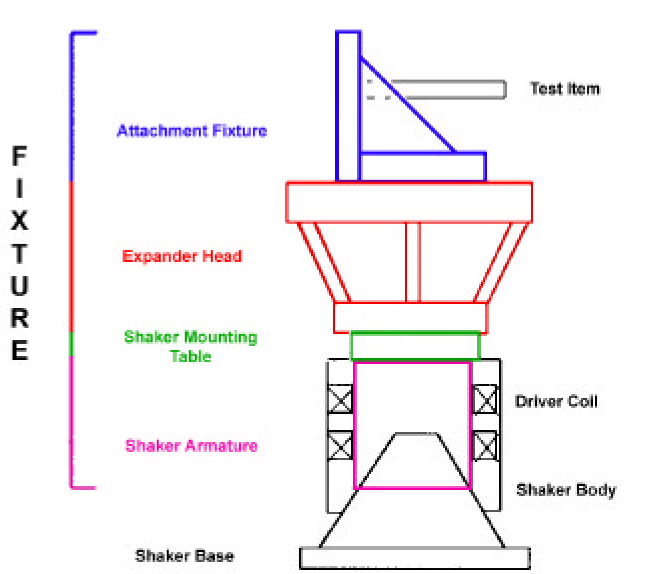

Figure 1. Typical fixture components.

Our ultimate goal would be to make this fixture infinitely stiff and massless. This essentially implies that the fixture would be resonance free, especially over the test frequency range of interest. This further implies that the surface to which the test article is attached moves as a rigid body with uniform displacement at the test article interface. This can be extremely difficult, if not impossible, to achieve, especially for larger shaker systems. These systems will generally have some resonant behavior due to the mass/stiffness characteristics of the armature, expander head (or slip plate) and attachment fixture to accommodate the test article. Many times, the test engineer ignores this basic problem and assumes that the shaker system controller will compensate for these effects.

Unfortunately, the controller cannot eliminate the resonances in a fixture, contrary to popular belief. The controller only maintains the level of acceleration at the location where the control accelerometer is mounted. The resonances of the system are not affected by the control feedback loop of the controller. All the feedback system does is adjust the input to the shaker’s driver coil, either up or down, to accommodate the specified level of acceleration desired. Usually, several accelerometers will be mounted at various points on the fixture and averaged together for the control signal. Again, this only maintains the average acceleration as a control parameter. The resonant frequency and dynamic characteristics are not changed. Multipoint averaging is an excellent technique for achieving a “better looking” control spectrum, but this does not change the resonant problems associated with the fixture.

What needs to be understood here is that the control accelerometer must be mounted on a surface that moves as a rigid body with no resonant characteristic over the frequency range of interest. This is what the controller expects to see. If this mounting interface is flexible, then the response of the control accelerometer will be heavily dependent on the location where it is mounted. Using several control accelerometers, averaged together, only implies that some average acceleration level is controlled. The only way that several control accelerometers can be correctly used on a flexible surface is if each of the control accelerometers has an independent feedback to a separate shaker which is controlled. Of course, this may be extremely difficult, if not impossible, to achieve since it requires multiple shakers and a MIMO type controller.

This may be confusing to many test engineers since they have been led to believe that the control accelerometer is all they need to be concerned about when performing vibration tests. But, this is not the case!

In order to understand some of these problems, some basic vibration concepts and theory need to be reviewed along with some examples to illustrate some of these problems. (There has been a significant amount of work performed to better understand the dynamic interaction between a test article and fixturing when going from field collected data to laboratory tests. Much of this work is related to the material here and has been documented in Reference 1.)

When presented with some of these unfamiliar concepts, many people often remark that this “is just an academic thing” and that “this doesn’t really happen,” Be assured that what is about to be described, does happen and is real. First let’s review the phenomena, and then let’s make rational engineering judgments as to how to best handle the situation.

The fact is that, in some cases, it is just not possible to have resonant free fixtures. However, it is very important for the test engineer to fully understand and to document the dynamics of the entire setup in order to conduct meaningful vibration tests. For example, if resonances do exist, then a good understanding of the mode shapes will help to either modify the fixture raising the resonances above the frequency range of interest or to mount the test articles to minimize the effects of these unavoidable resonances.

Now let’s discuss the theory which dictates why this happens, illustrated by some examples to show what can happen. (The next sections are not for the faint of heart. You may not like what you are about to be told!)

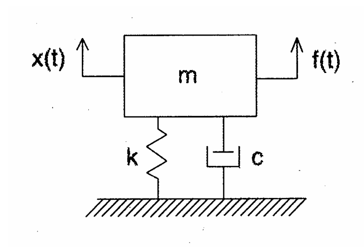

Figure 2: Single degree of freedom (SDoF) system

Basic Theoretical Concepts

The majority of the equations shown here can be found in any textbook on vibration. Most of the equations shown here were taken from Reference 2. The first basic equation is that of a single degree of freedom (SDoF) system shown in Figure 2. The equation of motion for this SDoF system is:

$$m\ddot{x} + c\dot{x} + kx = f\left(t\right)$$

Without going through all the steps involved, recall the equation describing the amplification for an SDoF system due to sinusoidal excitation as given by:

$$\frac{x}{\delta_{st}} = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}}$$

This equation is written as the ratio of the dynamic displacement to the static displacement. Note that the drive sinusoidal frequency is ? , the natural frequency is ?$_n$ and ? is the percent of critical damping. Looking at this equation, we see that the amplification ratio increases and then decreases as we sweep past the natural frequency of the SDoF system as shown in Figure 3.

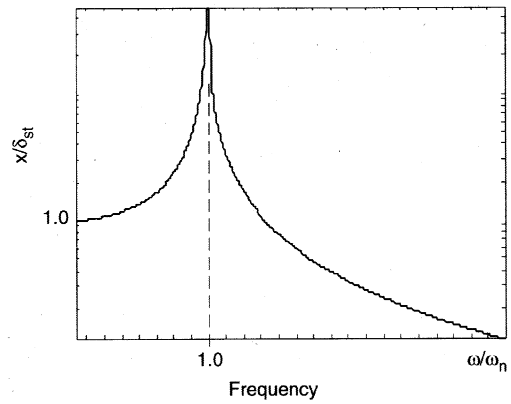

Figure 3. SDoF amplification due to sinusoidal excitation

Now, a system that is described by more than one mass interconnected by massless springs is a multiple degree of freedom (MDoF) system which is the next level of approximation. Since many equations result when more and more masses are included in the description of the system, matrices are used to describe the equation of motion. First let's look at a two DoF system and then generalize to more DoF.

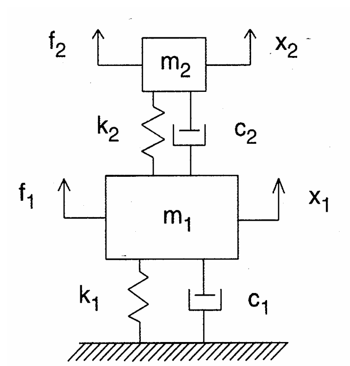

Figure 4. Multiple (two) degree of freedom system

Using a two mass system shown in Figure 4, the equation of motion can be written as:

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = f_1(t)$$

or in matrix form as

$$\begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$$

$$+\begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1(t) \\ f_2(t) \end{Bmatrix}$$

or

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\}$$

Using this form, any number of equations or masses can easily be handled. Now in this form, the equations exhibit coupling between the different DoFs. We use a mathematical process, called an eigensolution, to break up these more complicated coupled equations into a set of very simple SDoF systems. When this is done, the equations that result are very simple, as shown:

$$\begin{bmatrix} \bar{m}_1 & & \\ & \bar{m}_1 & \\ & & \ddots \end{bmatrix} \begin{Bmatrix} \ddot{p}_1 \\ \ddot{p}_2 \\ \vdots \end{Bmatrix} + \begin{bmatrix} \bar{c}_1 & & \\ & \bar{c}_2 & \\ & & \ddots \end{bmatrix} \begin{Bmatrix} \dot{p}_1 \\ \dot{p}_2 \\ \vdots \end{Bmatrix}$$

$$+ \begin{bmatrix} \bar{k}_1 & & \\ & \bar{k}_2 & \\ & & \ddots \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \\ \vdots \end{Bmatrix} = \begin{Bmatrix} \{u_1\}^T \{F\} \\ \{u_2\}^T \{F\} \\ \vdots \end{Bmatrix}$$

Each equation is uncoupled from every other equation and each basically describes a SDoF system corresponding to each of the natural frequencies of the system. So, we have taken a complicated set of equations and broken it up into a number of much simpler equations. These are shown schematically in Figure 5.

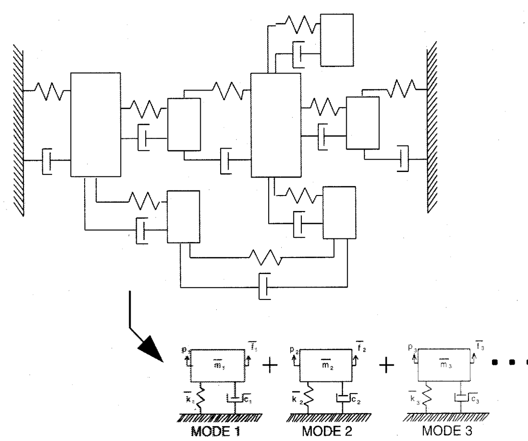

Figure 5. Uncoupling of MDoF system.

This implies that the SDoF theory also applies to the MDoF system for each of the modes of vibration of the MDoF system. So now look at the frequency response for the system shown in Figure 6 as nothing more than the sum of a set of SDoF systems.

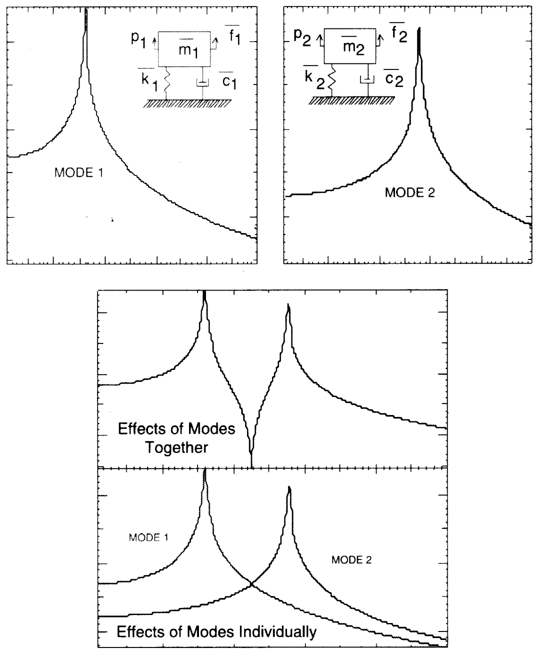

Figure 6. Mode superposition of each of the SDoF systems

If we can think about the relationship between a fixture and a test specimen as the dynamic relationship between each of the modes of each of the systems, then some simple equations can be utilized to understand dynamic coupling between the fixture and test article. Let's recall the equation for a two DoF system for the characteristics of a tuned absorber. This is a very familiar equation found in all vibration textbooks.

$$\frac{x_1 k_1}{F_0} = \frac{\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]}{\left[\left[1+\frac{k_2}{k_1}-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]-\frac{k_2}{k_1}\right]}$$

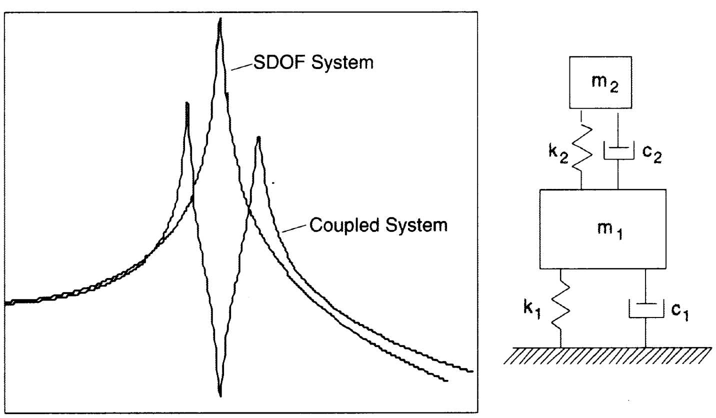

Figure 7. Tuned absorber characteristics.

The form shown here was obtained from Reference 3. This equation identifies that there is a relationship between the natural frequency and stiffness ratio of each of the separate single DoF systems. The amount of dynamic interaction is heavily dependent on the relative ratio of these terms. For a given mass ratio with the two single DoF systems having the same natural frequency, a typical plot of the final dynamic characteristics is shown in Figure 7. The amount of separation between these two peaks of the coupled system is dependent on the relative mass ratios.

Now we can use this equation for designing tuned absorbers for problem resonances, for designing seismic masses, etc. But this equation also helps to describe the dynamic interaction between any two systems, if we characterize them as SDoF systems. (Recall that any complicated MDoF system can always be represented as a set of simple SDoF systems.)

Simple Fixture Consideration

In order to demonstrate some of the dynamic coupling effects discussed, let's consider a representation of a vibration fixture and test specimen using a simple model to illustrate some important points.

Let's consider the use of two different test fixtures to be used with a test specimen. A simple two DoF model is used to represent the test article and a simple single DoF model is used to describe the first resonant frequency of each of the test fixtures. This is schematically shown in Figure 8.

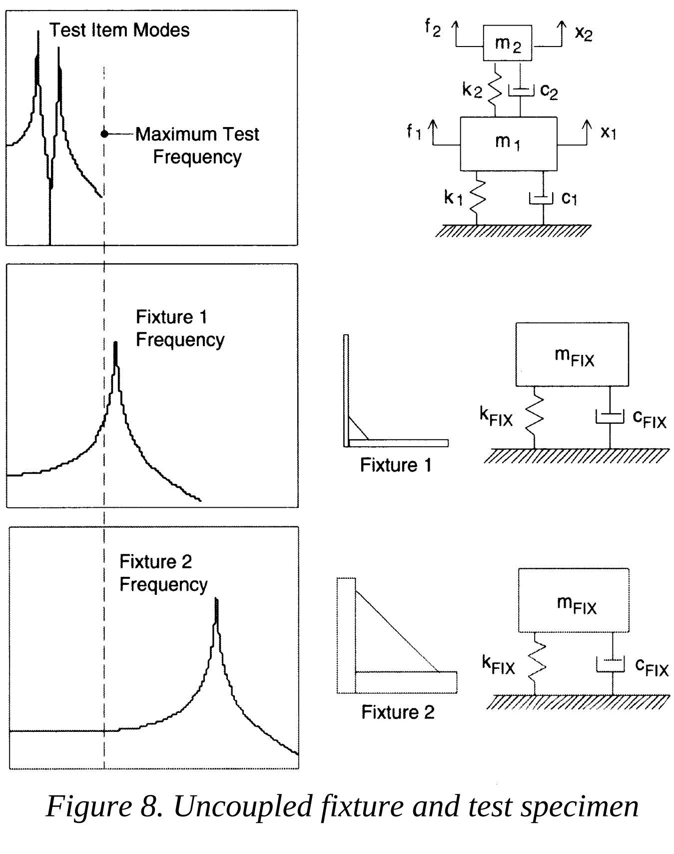

Figure 8. Uncoupled fixture and test specimen

For this example, the test article is to be tested up to the maximum frequency shown. Both fixtures that are considered for the test have resonant frequencies beyond the test frequency range but fixture 1 has its first resonant frequency very close to the test range upper frequency.

Now the systems are coupled together, and the resulting frequency response is shown in Figure 9 for the two fixtures.

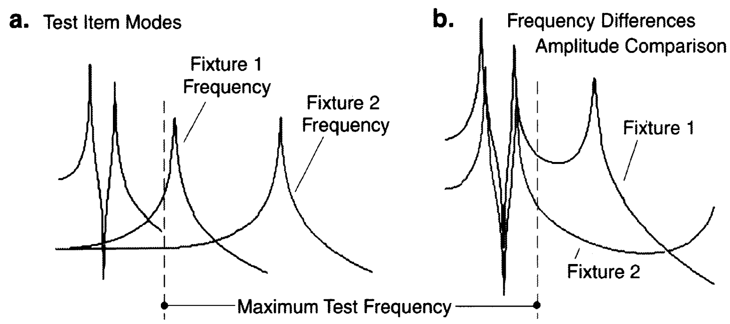

Figure 9. (a) Uncoupled fixture test specimens. (b) Coupled fixture test specimens.

The first thing to note is that the resonant frequencies of the test specimen are only slightly different comparing the two different fixtures. However, the amplitude of the response is significantly different. This means that the response levels are different. (Without going into all the detailed theory, we know that these amplitudes are directly related to the mode shape of the system at each particular frequency.) So while the frequencies are not changed significantly, the mode shapes (or indirectly, the response levels) are significantly different. This means that the test specimen will be exposed to different levels of acceleration than desired due to the use of the flexible fixture.

This example clearly shows that the fixture should be as stiff as possible and should be well beyond the test frequency range of interest otherwise dynamic coupling between the test article and fixture may result.

Expander Head Example

The last example showed that even with a fixture resonant frequency beyond the test range of interest, there can be some dynamic interaction between the test specimen and fixture. But what if the fixture has a resonant frequency in the test range of interest. Then what?

To illustrate some important points, an analytical model of a rib-stiffened expander head with a simple frame type test article was studied. The expander head was analytically modeled to have resonant frequencies in the test frequency range. If the expander head were infinitely stiff and resonance free, then it wouldn’t matter where the test specimen was mounted on the expander head (nor where the control accelerometer was mounted). So for this case, two models were studied — one with the test specimen symmetrically mounted to the expander head and the other with it asymmetrically mounted to the expander head. If the expander head is resonance free, then it shouldn’t matter where the test specimen is mounted (or where the control accelerometer is mounted).

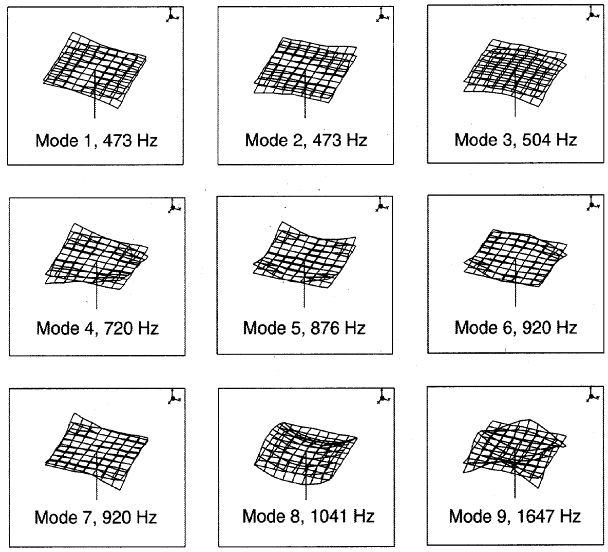

Figure 10. Bare expander head mode shapes.

For reference, the first nine modes of the bare expander head are shown in Figure 10. Note that none of the rocking modes about the symmetry axes would be excited by a uniaxial input. These modes are shown here for completeness and reference.

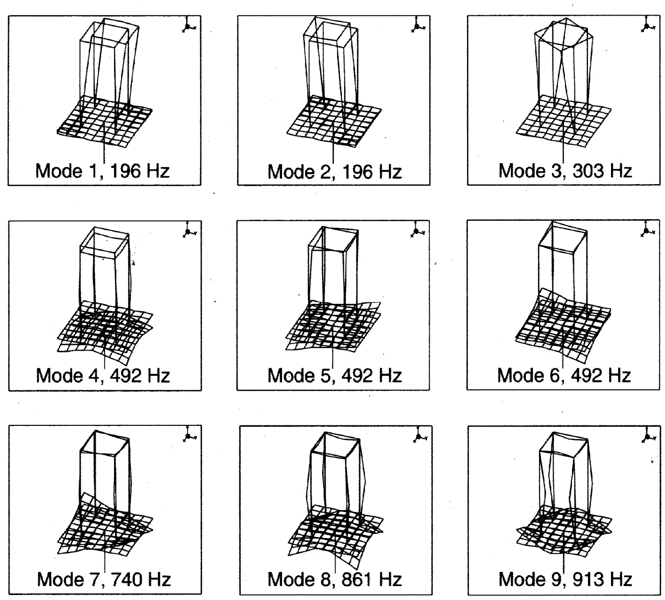

Figure 11. Mode shapes for test specimen symmetrically mounted on expander head

The first nine modes of the expander head with the test specimen symmetrically mounted on the expander head are shown in Figure 11. Again, none of the rocking modes about the symmetry axes would be excited by a uniaxial input. Notice that all of the mode shapes are symmetric for this case with the frame mounted symmetrically on the expander head.

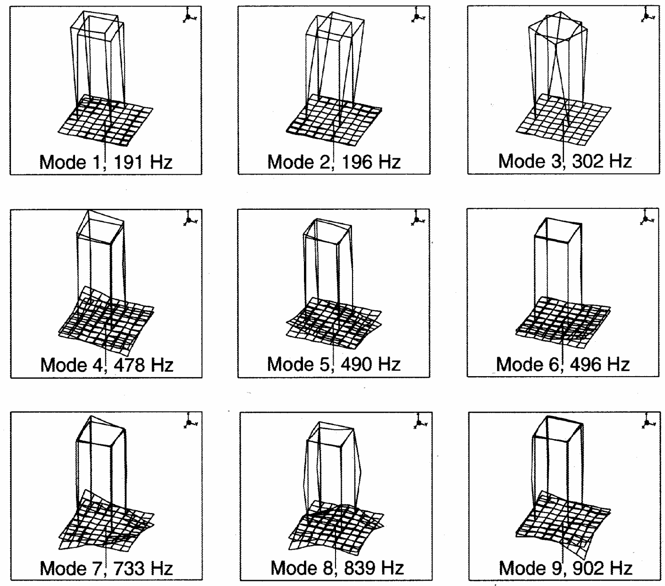

Figure 12. Mode shapes for test specimen asymmetrically mounted on expander head

The first nine modes of the expander head with the test specimen asymmetrically mounted on the expander head are shown in Figure 12. Here all of the mode shapes are not symmetric for this case with the frame mounted asymmetrically on the expander head. Now it is very important to notice that the mode shapes of the test specimen are significantly different between the symmetric and asymmetric mount conditions shown in Figures 11 and 12.

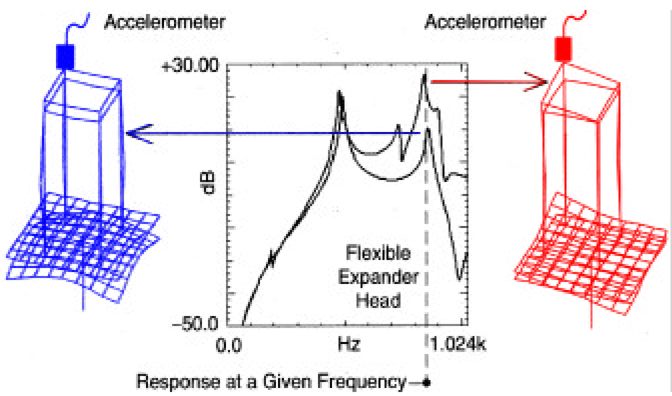

Figure 13. Symmetrically/asymmetrically mounted test specimen on resonant expander head.

If we were to perform a simple sweep and compare the response at one of the frequencies, significant differences would be seen as shown in Figure 13. Now if we look at the response at one frequency, we can see differences, especially around the top of the frame structure. The symmetric mount graphic shows the top of the frame moving with the same amplitude. The asymmetric mount shows the back corner to have significantly different amplitudes from the rest of the structure. Obviously, there are significant differences between these two setups.

Now let's think about where the control accelerometer should be located. It's not clear where it should be mounted for this test, since the expander head is not rigid at all frequencies. There could be a serious overload or underload of the test specimen depending on the location selected.

What if the control accelerometer was mounted on the structure itself at the top of the frame? There are significant differences at the top of the frame for the asymmetrically mounted configuration. Naturally, the control accelerometer typically would not be mounted at this location.

But what if the frame were another intermediate fixture that was mounted on the expander head to accommodate mounting of test articles? Would the dynamics of the frame/fixture have an effect on the test to be performed? The answer is “YES — definitely!”

We can see that the dynamics of the fixture can affect the results of the test to be performed. The control accelerometer can only adjust the input signal to the armature up or down, to maintain the level desired. The control accelerometer cannot change the dynamic situation that exists based on the coupling between the expander head and test article.

Of course, it is very important to realize that the fixture is not just the fixture we design to accommodate the test article. It is the expander head (or slip plate), the armature and the intermediate fixtures we design to accommodate test hardware. The dynamic characteristics of all of these items come into consideration when performing a vibration test. If any of these components has resonances in the frequency range of interest, then problems may exist.

It is very important to understand that this resonant situation may exist. The results of a finite element model or an experimental modal test are useful in identifying these potentially troublesome frequencies.

Armature as a Fixture

Up until this point, only the physically exposed parts of the shaker system have been discussed. However, even the bare shaker armature may have resonances below 2000 Hz. This is especial1y true for larger shaker systems with armatures over 20 in. in diameter. Sometimes the armature itself will have more than one resonance below 2000 Hz.

These resonances may cause problems with the test and often will first appear as a shaker control problem. The test engineer immediately blames the control system for all the problems, claiming that the “control system doesn’t function properly.” If an expander head is included in the shaker system, it is often also blamed for the problems at hand. It is rare that the test engineer would even consider the possibility that the mechanical armature might be part of the problem. Yet, many times this is the case. If the armature has resonances, then the articles mounted to the mounting head of the shaker may be affected by these resonances.

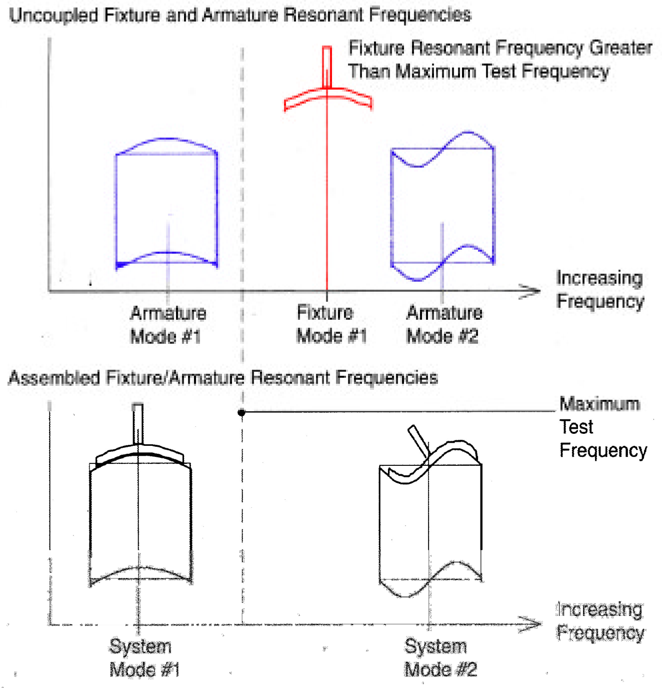

Figure 14. Fixture/armature assembly considerations.

One simple illustration of the armature/fixture problem is shown in Figure 14. A relatively stiff, well designed fixture is mounted to a flexible armature that has resonances in the frequency range of interest. While the fixture may be resonance free (with resonant frequencies well beyond the frequency range of interest), the system must be considered in its final form. When the rigid fixture is mounted on the flexible armature, the fixture will take on a deformation pattern resulting from the assembled system. So we can see that the fixture is affected by the armature. Together they form a system that must be considered as a whole. It is difficult (if not impossible) to design a fixture without including the significant dynamics of the attached armature.

As discussed before, the control accelerometer location becomes an issue for the case illustrated in Figure 14. The location where it is mourned along the deformed, flexible surface will have a significant effect on the test performed.

Another fallacy is that many believe that multiple control accelerometers will correct this problem. This is completely untrue. The use of multiple accelerometers for control only implies that some “average” acceleration value is used for the feedback to the control system.

The solution to this problem clearly rests with the redesign of the armature to eliminate the resonances causing the problems, But no one wants to actually implement this type of change due to the cost involved in designing a large shaker armature that is resonant free for the frequency range of interest. This can be a significant task to achieve. Also, no one wants to be the “whistle blower” in the test lab or at the shaker manufacturing facility to identify that the armature resonances, first of all, do exist, and secondly, do have an effect on the tests performed. But this is the reality of the situation!!!

So What Can I Do???
First, let's absolutely state that the solution to the problem is clearly to design resonant free fixtures for all vibration tests. Remember that the fixture is the armature, expander head (or slip plate), intermediate fixtures, etc. This is the solution to the problem!

But we also recognize that to design an infinitely stiff, resonant free, massless fixture is impossible. Therefore, as engineers, we need to be practical. If possible (and sometimes it is not possible), we need to design effective solutions to problems. At the very least, we need to identify an effective "work around." We also need to carefully and fully document the situation.

An armature (or expander head) typically may have only one or two resonances that are troublesome. These need to be identified so that some "work around" or "compromise" can be developed. Finite element models can be used in the early stage of the design of a fixture system to lend insight into weak areas of the design. For existing systems, the results of an experimental modal survey will often identify the resonant frequency and mode shape associated with the fixture assembly. The animated mode shapes often vividly identify the troublesome areas. This helps in the correction of problem frequencies that may exist.

The point here is to clearly identify the problem so that engineering judgment can be used to solve the problem or identify alternate solutions or "work arounds." Of course, ultimately, we should always strive for resonant free fixture assemblies rather than applying "band aids" to fix the problems.

Summary
Some basic dynamic test considerations for vibration fixture design were discussed. Resonances and dynamic coupling effects were discussed as to their importance to the vibration test. These effects may be important to the overall response of the system. Several examples were used to highlight the potential problems when using resonant fixtures for vibration tests.

Vibration fixtures (armatures, expander heads, slip plates, interface fixtures) are an important part of the vibration test system. Consideration needs to be given to all the elements of the fixture. They all need to be resonant free over the test frequency range of interest.

When resonances exist in the setup, their effects cannot be removed through the use of feedback from single or multiple control accelerometers. Resonances must be fully documented and understood for the entire vibration setup. Then engineering judgment can be used for the situation at hand so that the best possible test can be conducted.

Resonances have a tremendous impact on vibration tests conducted. Resonances don't disappear just because no one took the time to investigate them. Nor do they disappear through the use of a controller system. What you see is what you get (even if you don't look).

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References

1. Varoto, P. S., “The Rules for the Exchange and Analysis of Dynamic Information in Structural Dynamics,” Ph.D. Thesis, Iowa State University, Ames, IA, 1996.
2. Avitabile, P., “Overview of Experimental Modal Analysis Using the Frequency-Response Method,” Lecture/Reference Notes, 1995.
3. Thompson, W. T., *Theory of Vibrations With Applications*, Prentice Hall, 2nd Edition, 1981.
4. Avitabile, P.,”Practical Aspects of Vibration Fixture Design,” Seminar Notes, 1996.