Test Time Exaggeration Factor

by George Hieber

Vibration tests should prove the durability of a device to perform properly throughout its expected life.

If a device is expected to last for hundreds or thousands of hours of use, the problem for the test engineer is to devise a way to evaluate the long term durability with a short term test. A method is sought which would cause the same potential damage in a short time as would be caused by the field environment throughout the life of the device.

It is natural to turn to the S-N fatigue curve in such a situation. This curve shows that the higher the peak stress, S, the fewer stress reversals, N, occur before fatigue failure. Since the critical response of an item is at a resonant frequency, f_r, the time to failure is $T = N/f_r$. The S-N curve can thus be used as an S-T curve. It is therefore possible to either choose a stress level increase and determine the resulting reduced time to failure, or to choose a desirable time to failure and determine the required stress increase. Incidentally, the word “failure” here can be construed to mean “equivalent damage.”

Since the test engineer is concerned with magnitude of input excitation, but the governing factor is stress, it is necessary to know the relationship between excitation level and stress. In order to evaluate this, the effective structural damping must be known—doubling the input excitation does *not* double the stress. As structural damping increases with stress, a doubling of the input excitation level will cause an increase of the effective damping ratio, with the result that the response may increase by considerably less than by a factor of two.

On the next pages, formulas are derived which are used to generate a test time exaggeration factor.

The factors involving stiffness (stored energy) and damping (dissipated energy) must be taken into account: *

The stored energy (U) in a structure is equal to the strain energy, which is proportional to the stress (σ) squared:

$$U = k_1\sigma^2$$

where k_1 is a factor taking into account the structural geometry.

The energy dissipated (D) per cycle of motion is proportional to a power of stress:

$$D = k_2\sigma^n$$

where k_2 is a material property. For pure viscoelastic damping, $n = 2$, but in general, n can vary up to 8 or more.

Q is a notation which originated in electrical engineering. It stands for Quality Factor, which is a ratio comparing the energy storage capability of a system with the energy dissipated:

$$Q = \frac{\text{stored energy}}{\text{dissipated energy}} = \frac{k_1\sigma_o^2}{k_2\sigma_o^n} = k_3\sigma^{2-n}$$

the “o” subscript indicates output, or response, stress.

*Derivation ref: Curtis, Tinling and Abstein, *Selection and Performance of Vibration Tests,* Shock and Vibration Information Center, Dept. of Defense, 1971.

Test Time Exaggeration Factor, continued

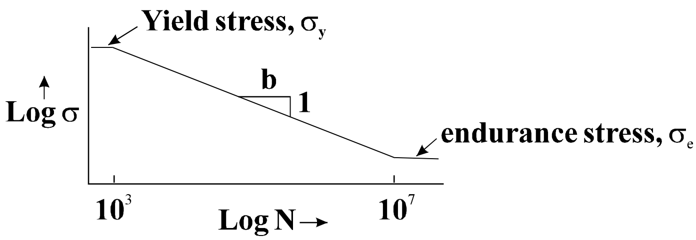
For sine excitation:

$$\text{Let } \sigma_o = k_4 G_{out}; \text{ then if } G_{out} = G_{in} Q$$

$$\sigma_o = k_4 G_{in} Q; \; G_{in} = \frac{\sigma_o}{k_4 Q} = \frac{\sigma_o}{k_4 \left(k_3 \sigma_o^{2-n}\right)}$$

$$\text{so } G_{in} = k_5 \sigma_o^{1-2+n} = k_5 \sigma_o^{n-1} \quad \text{or } \sigma_o = k_6 G_{in}^{\frac{1}{n-1}}$$

From fatigue S-N curve:



Between σ_e and σ_y, the equation for the S-N relationship is given by:

$$N\sigma^b = C, \text{ where C is a constant}$$

$$\text{so } N_1 \sigma_1^b = N_2 \sigma_2^b; \text{ but since } f_r T = N,$$

$$f_r T_1 \sigma_1^b = f_r T_2 \sigma_2^b \quad \text{or } T_1 \sigma_1^b = T_2 \sigma_2^b$$

$$\text{then } \frac{T_1}{T_2} = \left(\frac{\sigma_2}{\sigma_1}\right)^b = \left(\frac{\not{k}_6 G_{in_2}^{\frac{1}{n-1}}}{\not{k}_6 G_{in_1}^{\frac{1}{n-1}}}\right)^b = \left(\frac{G_{in_2}}{G_{in_1}}\right)^{\frac{b}{n-1}}$$

$$\text{so } T_2 = T_1 \left(\frac{G_{in_1}}{G_{in_2}}\right)^{\frac{b}{n-1}} \text{ or } G_{in_2} = G_{in_1} \left(\frac{T_1}{T_2}\right)^{\frac{n-1}{b}}$$

For random excitation:

The response of single degree of freedom system is:

$$G_{out}^2 = \frac{\pi}{2} W_{in} f_r Q = k_7 W_{in} f_r Q$$

$$\text{Let } \sigma_o^2 = k_8 G^2_{out}, \text{ then } \sigma_o^2 = k_8^2 k_7 W_{in} f_r Q$$

$$\sigma_o^2 = k_8^2 k_7 k_3 W_{in} f_r \sigma_o^{2-n} = k_9 W_{in} f_r \sigma_0^{2-n}$$

$$\text{Rearranging, } \frac{\sigma_o^2}{\sigma_o^{2-n}} = k_9 W_{in} f_r; \; k_9 W_{in} f_r = \sigma_o^{2-2+n} = \sigma_o^n$$

$$\text{and } \sigma_o = k_{10} \left(W_{in} f_r\right)^{\frac{1}{n}}$$

$$\text{Using the S-N relationship, } \frac{T_1}{T_2} = \left(\frac{\sigma_2^{\frac{1}{n}}}{\sigma_1^{\frac{1}{n}}}\right)^b = \left[\frac{\not{k}_{10} \left(W_2 \not{f}_r\right)^{\frac{1}{n}}}{\not{k}_{10} \left(W_1 \not{f}_r\right)^{\frac{1}{n}}}\right]^b$$

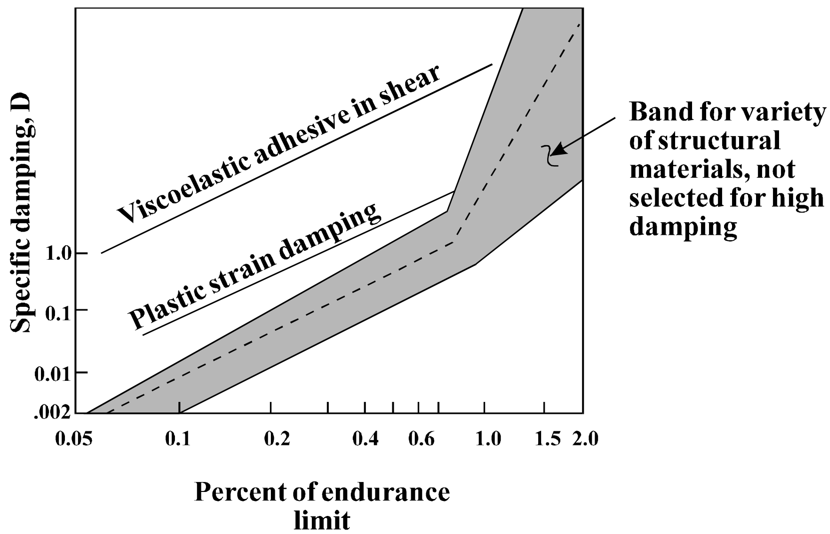
$$\text{so } T_2 = T_1 \left(\frac{W_1}{W_2}\right)^{\frac{b}{n}} \text{ or } W_2 = W_1 \left(\frac{T_1}{T_2}\right)^{\frac{n}{b}}$$

Test Time Exaggeration Factor, continued

The problem at this point is to select values for b and n.

A number of metallic structural materials exhibit an S-N slope close to 9, so frequently in books and papers on the subject, we see b = 9.

When it comes to damping, the subject is more complex. The values for damping usually come from a 1959 paper by B. Lazan in the ASME Colloquium *Proceedings*.



The dashed line follows the median values of the specific damping, D, as the stress is varied. Since $D = k_2\sigma^n$, the value of n can be obtained by choosing two points on the straight portions of the dashed line, getting the coordinates D_1, D_2, $\%\sigma_{e1}$ and $\%\sigma_{e2}$ and solving

$$n = \frac{\log \frac{D_1}{D_2}}{\log \frac{\sigma_1}{\sigma_2}}$$

Following this process, it is seen that at stresses below 80% of the endurance limit, n = 2.4. At higher stresses, n = 8.

The values chosen in present publications are b = 9 and n = 2.4.

This leads to:

$$\frac{b}{n-1} = \frac{9}{2.4-1} = 6.4$$

$$\text{and } \frac{b}{n} = \frac{9}{2.4} = 3.75$$

MIL-STD-810E, page 514.4-46 “rounds off” the above values to

$$M = \frac{b}{n-1} = 6 \text{ for sinusoidal excitation}$$

$$M = \text{and } \frac{b}{n} = 4 \text{ for random excitation}$$

Test Time Exaggeration Factor, continued

It is stated, vaguely, that "other values may be appropriate." It is also stated that M = 2.5 should be used for sinusoidal excitation of electronic boards. No references are given.

Although the equations for the determination of the test time exaggeration factor (TTEF) are correct and properly derived, their meaningful use depends on a logical choice of values for parameters b and n. Values for these parameters must be chosen with care, and yet, unfortunately, little guidance is available.

There are many occasions when the TTEF should not be used. If a device is designed such that the stress levels of critical parts in service are at or below the endurance level, the parts will never fail. As a matter of fact, this is the goal of most design. In this event, the use of a TTEF makes no sense.

Suppose a device with a design life of 1000 hours is to be tested, and it is desired to evaluate the endurance of the device, but the test duration is to be limited to one hour. If the test is a random vibration test, the TTEF equation as recommended in MIL-STD-810E calls for a test PSD level 5.6 times that of the field level. If this test were to be run, but a structural part fails in less than one hour, the question should be asked, "Is this a valid test result?"

The only way to tell is to determine what the ratio is between stress levels on the part during field and test environmental levels. This may require an instrumented test on a similar assembly. For instance, for the above example, if the value b = 9 is correct for this material, the ratio of RMS stress during test to RMS stress during field events should be approximately:

$$\sigma_{rms_T} = \sigma_{rms_F}\left(\frac{T_F}{T_T}\right)^{\frac{1}{b}} = \sigma_{rms_F}\left(\frac{1000}{1}\right)^{\frac{1}{9}} = 2.15\sigma_{rms_F}$$

If so, the test result is valid and the part should be redesigned, *unless* the σ_{rmsF} is at or below the endurance stress, in which case the test is probably invalid.

If the ratio is different from 2.15, however, the reason may be due to a poor choice of b, n or both.

The problem with the formulas in use is that the relationship $NS^b = C$ for structural fatigue is valid only from σ_e to σ_y, so a value such as b = 9 is applicable for stresses above σ_e and below σ_y, and yet the value for average damping, n = 2.4, is the structural damping at stresses *below* σ_e. Thus, the values chosen for b and n are not compatible.

These values may have been chosen to provide an "average" result for various excitations, but the fact remains that they are arbitrary and can lead to erroneous results.

For instance, if the value n = 5 were chosen (which seems to be more logical because of the higher damping with higher stress), the time duration required for the test would be:

M in the MIL-STD-810E formula becomes $M = \frac{b}{n} = \frac{9}{5} = 1.8$

Then $T_2 = 1000\left(\frac{1}{5.6}\right)^{1.8} = 45.8$ hours.

This means that if n = 5 is a better estimate of the damping exponent, the use of 2.4 in the formula, as done earlier, would lead to a gross *undertest*.

Test Time Exaggeration Factor, continued

As another example, if the critical part is made of a material which has an S-N slope significantly different from 9, large errors can occur in the estimate of a TTEF. For instance, if the material is 17-4PH stainless steel, the slope is b = 17.6

Using n = 2.4, $M = \frac{b}{n} = \frac{17.6}{2.4} = 7.33$

$$T_2 = 1000\left(\frac{1}{5.6}\right)^{7.33} = .003 \text{ hours} = 12 \text{ seconds}$$

Using n = 5, $M = \frac{b}{n} = \frac{17.6}{5} = 3.52$

$$T_2 = 1000\left(\frac{1}{5.6}\right)^{3.52} = 2.3 \text{ hours}$$

Compared to the one hour test duration obtained by using b = 9 and n = 2.4, it is apparent that *if* b = 17.6 and n = 2.4 are more accurate, the one-hour test would be a gross *overtest* and *if* b = 17.6 and n = 5 are more accurate, the one hour test would be a significant *undertest.*

Therefore, specific S-N data should be used, rather than average, and more experimental material damping data should be generated.